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Spreading and contraction at the boundaries of free turbulent flows

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The unstrained free turbulent flow generated by a composite grid has been found to possess a distinct, deeply indented boundary separating non-turbulent fluid from turbulent fluid of virtually homogeneous intensity. At this boundary, contraction, in the sense of a reduction in the volume of turbulent fluid, takes place at a rate which is independent not only of the intensity and degree of anisotropy of the turbulence, but also of the relative depth of the surface indentations. The apparent outward spreading of the turbulence is solely due to the increasing amplitude of the boundary bulges.

The subsequent plane straining of the flow in a constant area distortion shows that a reasonably high degree of anisotropy, combined with straining by the mean flow, is required before entrainment of non-turbulent fluid can occur at the boundary. Under these simple straining conditions, the rate of growth of the volume of turbulent fluid appears to be independent of the relative depth of indentations in the turbulence front.

It is considered that in free turbulent shear flows, the potential-turbulent flow boundary will advance into the non-turbulent fluid at a rate which depends on the local mean rate of strain and the local degree of anisotropy adjacent to the front. Contraction is expected to occur on those occasions when the boundary lies in a region of zero mean rate of strain, or when the anisotropy of the turbulence is low. The increased rate of spread due to large eddy activity is thought to be mainly caused by an increase in the probability of the potential-turbulent flow boundary being in a position where the mean rate of strain and local anisotropy are both high, rather than being entirely due to an increase in the surface area of the front.

1. Introduction

The distinctive feature of free turbulent shear flows is the existence of a sharply defined boundary separating fluid in turbulent motion from the surrounding potential flow. The mechanism of entrainment of non-turbulent fluid by turbulent fluid across this bounding surface is the basic process in the spreading of jets and wakes, and is far from being completely understood.

The interface is highly indented, apparently due to the influence of eddies having a scale an order of magnitude larger than that of the main body of the turbulence, and this folding of the bounding surface is considered by Townsend (1956, 1966) to play a fundamental part in the entrainment process. Townsend's equilibrium hypothesis is based on the assumption that the large eddies derive their energy from the mean flow and that their growth is resisted by the small-scale turbulence absorbing some of their energy. A control cycle is suggested in which growth of the large eddies results in increased indentation of the bounding surface and rapid entrainment. This in turn leads to an increase in the turbulent intensity and additional damping of the large eddies. Observations by Grant (1958) of the large eddies in a plane wake behind a circular cylinder suggest an oscillation of the control cycle. The large eddies tend to occur in groups, with periods of quiescence and weak entrainment alternating with periods of large eddy activity and rapid entrainment. Gartshore (1966) has obtained confirmation of the equilibrium hypothesis from measurements in a number of self-preserving wall jets.

The entrainment process at the interface has been considered by Corrsin & Kistler (1955). They concluded, by dimensional reasoning, that in the absence of mean flow shear, the velocity at which the turbulence front propagates into the non-turbulent fluid should be proportional to $(\nu\omega)^{\frac{1}{2}}$, where ω' is the root mean square of the vorticity, and that in general it will also depend on some characteristic mean shear stress. A fundamental approach to this aspect of the entrainment problem would involve the study of the spreading of a semi-infinite field of homogeneous turbulence into adjacent non-turbulent fluid, in the absence of mean velocity gradients. This has been attempted by Corrsin & Kistler (1955). The method consisted of covering half the wind-tunnel cross-section with a conventional turbulence generating grid, while the other half was covered by a fine mesh gauze of identical static pressure drop. The attempt was unsuccessful due to complexities arising in the flow around the joint between grid and gauze.

However, the successful production of a free turbulent flow without mean velocity gradients, using a composite grid, has been described by Townsend (1956). The grid consists of two identical wake-producing elements, joined to the tunnel roof and floor by fine mesh gauze. Immediately downstream of the grid, the flow consists of a jet flanked by two wakes. When the spacing between the wake producing bars is carefully adjusted, the mean pressure drop over the composite grid can be made equal to that of the gauze. The mean velocity variations then disappear within a small distance of the grid. The axi-symmetric equivalent of this flow has been studied by Naudascher (1965).

Turbulence intensity measurements have given rise to the assumption that the turbulence created by the composite grid is inhomogeneous with plane symmetry, and is in process of diffusing outwards. The present investigation, which includes measurements of intermittency factor, shows that the unstrained turbulence is almost homogeneous and possesses a sharp, irregular boundary similar to that in free turbulent shear flows. Consequently there exists the opportunity of studying an interface between non-turbulent and virtually homogeneous turbulent fluid in the absence of mean velocity gradients.

Reynolds (1962) and Keffer (1965) have investigated the effect of applying a plane strain to the two-dimensional wake behind a circular cylinder using a constant area distorting duct. Additional information concerning the character of the laminar-turbulent interface behind the composite grid has been obtained by plane straining using a similar constant area distortion. Unfortunately the maximum strain ratio of this duct was only 2 compared with a maximum ratio of 4 obtained by Reynolds and Keffer, and this may be insufficient to produce a full response of the turbulence to strain.

The effect on the interface of a sudden removal of the strain has also been observed.

2. Experimental equipment and procedure

The arrangement of the composite grid is shown in figure 1. The wind tunnel has a 1 ft. square working section, normally 4 ft. long; but for the present experiments an additional 4 ft. length of working section was added. The composite grid is located at the start of the working section, and is composed of two $\frac{1}{4}$ in. square elements joined to the roof and floor of the tunnel by fine mesh gauze.

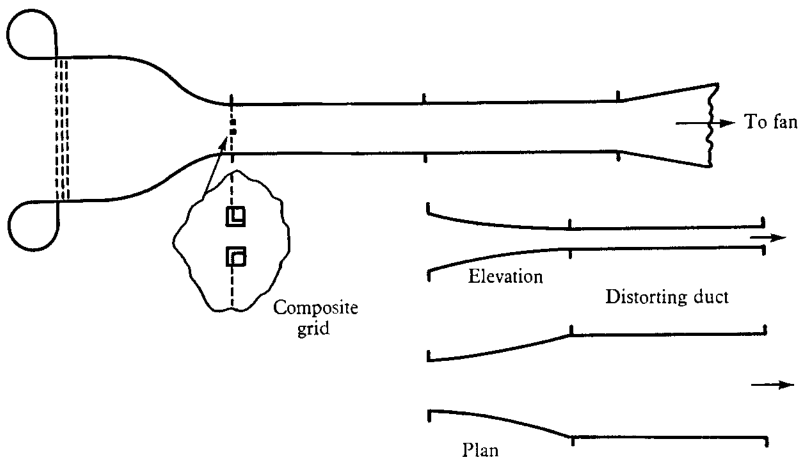

FIGURE 1. General arrangement of wind tunnel and distorting duct for the composite grid experiment.

Measurements of mean velocity were made with a pitot tube at stations 38, 67, 104, 127, 160 and 189 cm from the grid. By very careful adjustment of the spacing between the $\frac{1}{4}$ in. square elements it proved possible to reduce the mean velocity variations to within experimental error from 104 cm outwards. Particular care was found to be necessary in ensuring that the grid was symmetrical with respect to the tunnel roof and floor, since very slight departures from this ideal were detectable as a lack of symmetry in the mean velocity distribution downstream of the grid.

At a later stage the second 4 ft. length of working section was replaced by a constant area distorting duct, having a maximum strain ratio of 2, and similar in design to the duct used by Townsend (1954) for the plane straining of homogeneous turbulence. Following the distortion was a length of parallel sided duct in which the effect of removing the strain could be studied.

Measurements of the turbulent intensities $\overline{u^2}$, $\overline{v^2}$ and $\overline{u^2}$, and of the mean square of the velocity derivatives $\overline{\left(\frac{\partial u}{\partial t}\right)^2}$, $\overline{\left(\frac{\partial v}{\partial t}\right)^2}$ and $\overline{\left(\frac{\partial w}{\partial t}\right)^2}$ were made using DISA single wire and X-probes in conjunction with DISA 55A01 Constant Temperature Anemometers and a DISA 55A06 Random Signal Indicator and Correlator. These results were used to determine values of the parameters

$$K_1 = \frac{\overline{v^2} - \overline{w^2}}{\overline{v^2} + \overline{w^2}}$$

and

$$K_1' = \frac{\overline{\left(\frac{\partial v}{\partial x}\right)^2} - \overline{\left(\frac{\partial w}{\partial x}\right)^2}}{\overline{\left(\frac{\partial v}{\partial x}\right)^2} + \overline{\left(\frac{\partial w}{\partial x}\right)^2}}$$

introduced by Townsend (1954) as measures of the degree of anisotropy of the energy containing eddies and the small scale dissipating eddies, respectively, in homogeneous turbulence subjected to a plane strain.

The intermittency factor γ was determined by the method of Corrsin & Kistler (1955). A circuit was used whose output is zero when the probe is in non-turbulent fluid and has a constant value when the probe is in turbulent fluid. The resulting on–off signal is then used to gate a high-frequency pulse signal on its way to a digital counter. Some checks were made on the accuracy of the method by directly measuring γ from oscillograms. In the absence of mean flow shear it might have been expected that the boundary between turbulent and non-turbulent fluid would become diffuse and difficult to detect. In fact it remained quite sharp and no difficulty was encountered in measuring the intermittency factors.

The distributions of intermittency factor were used to calculate the mean position of the interface

$$\overline{Y} = \int_{-\infty}^{\infty} y \frac{\partial \gamma}{\partial y} dy.$$

As a measure of the width of the intermittent zone a length,

$$L_\gamma = \left[\frac{\int_{-\infty}^{\infty} \gamma y^2 dy}{\int_{-\infty}^{\infty} \gamma dy}\right]^{\frac{1}{2}},$$

was chosen in preference to the more usual standard deviation,

$$\sigma = \left[\int_{-\infty}^{+\infty} (y - \overline{Y})^2 \frac{\partial \gamma}{\partial y} d(y - \overline{Y})\right]^{\frac{1}{2}},$$

for reasons which will be developed later.

3. Results

3.1. *The unstrained turbulence*

The flow immediately downstream of the composite grid is a complex free-shear flow consisting of a jet flanked by two identical wakes. During this phase, spreading of the turbulence is taking place, and the mean position of the turbulence front moves outwards from the centre line of the flow (figure 2). At a distance of 104 cm from the grid the mean velocity variations have virtually disappeared and the turbulence commences to contract, with $\overline{Y}$ decreasing at a linear rate.

At about the same distance from the grid the increase in the width of the intermittent zone L_γ also becomes linear with increase in distance from the grid (figure 3). The combination of contraction of the turbulence and increasing width of the intermittent zone soon leads to values of intermittency factor γ less than unity at the centre of the flow (figure 4). The increase in width of the inter-

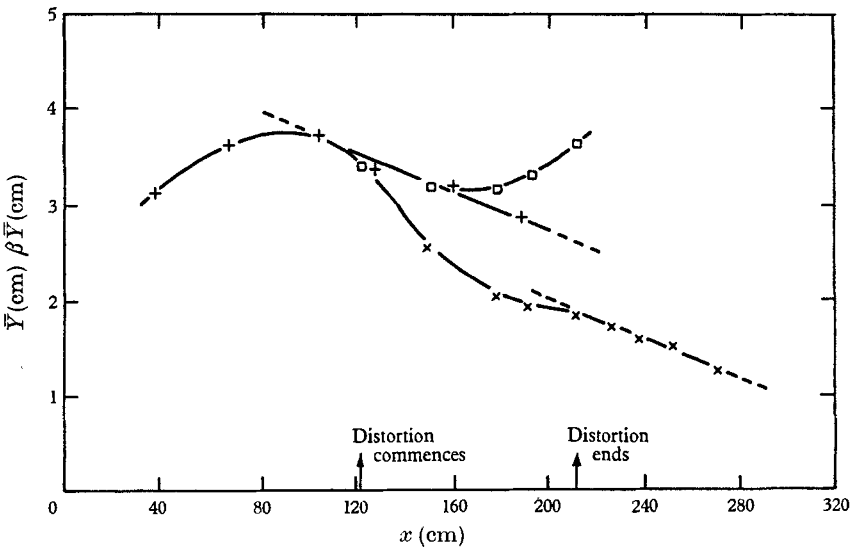

FIGURE 2. Variation of mean turbulence boundary position $\bar{Y}$ with distance x from the composite grid. +, undistorted turbulence, $\bar{Y}$; ×, distorted turbulence, $\bar{Y}$; □, distorted turbulence, $\beta\bar{Y}$.

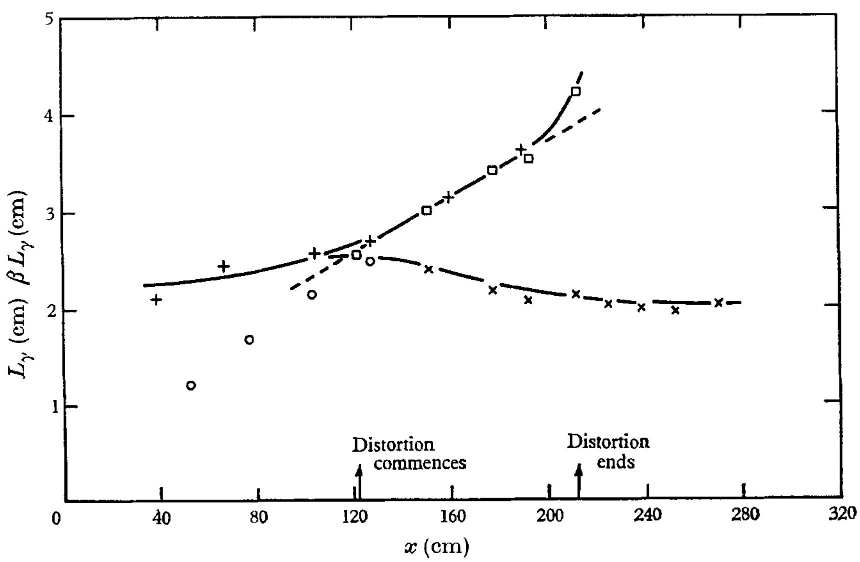

FIGURE 3. Variation of the scale of the intermittent zone L_γ with distance x from the composite grid. +, undistorted turbulence, $\bar{Y}$; ×, distorted turbulence, $\bar{Y}$; □, distorted turbulence, $\beta\bar{Y}$; ○, values of Townsend's scale L (Townsend 1956).

mittent zone is sufficient, in spite of the contraction of the turbulence, to cause turbulent fluid to appear at increasing distances from the centre line, and it is this which has previously lead to the assumption, based on intensity measurements, that the turbulence spreads outwards.

Within the turbulent fluid, the turbulent intensity is approximately $\overline{q^2}/\gamma U^2$. In figure 5 this quantity is seen to become progressively more uniform with distance from the grid, showing that the variations in the measured turbulent intensity are mainly due to intermittency factor variations. The intensity ratios $\overline{u^2}/\overline{v^2}$ and $\overline{v^2}/\overline{w^2}$ on the centre line of the flow (figure 6) indicate a gradual approach to isotropy, although complete isotropy is not obtained with the experimental range.

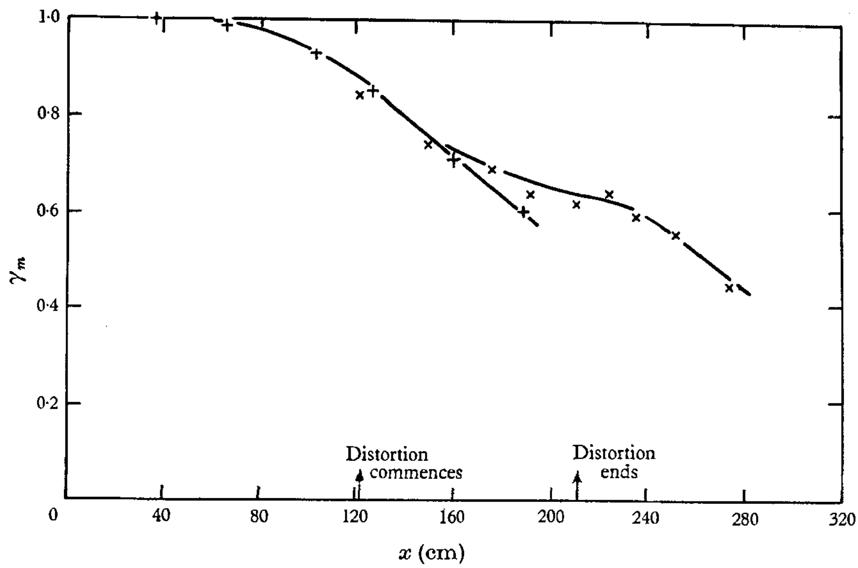

FIGURE 4. Variation of intermittency factor on the centre line of the flow γ_m with distance x from the composite grid. +, undistorted turbulence; ×, distorted turbulence.

Assuming uniform turbulent intensities within the turbulent part of the flow, the scale

$$\bar{L} = \left[\frac{\int_{-\infty}^{\infty} \overline{u^2} y^2 dy}{\int_{-\infty}^{\infty} \overline{u^2} dy}\right]^{\frac{1}{2}}$$

used by Townsend (1956) to define the width of the inhomogeneity in the measured values of $\overline{u^2}$, should correspond with L_γ. Townsend's values for L are included in figure 3 where it appears that they eventually approach the same slope as L_γ.

Townsend showed that the distributions of $\overline{u^2}$ and $\overline{v^2}$ became approximately self-preserving. The intensities were non-dimensionalised using a velocity u_0 defined by

$$u_0^2 = L^{-1} \int_{-\infty}^{\infty} \overline{u^2} dy.$$

If we assume the intensities to be uniform within the turbulent fluid then $\overline{u^2}/u_0^2 = \gamma L_\gamma/2\overline{Y}$. A plot of $\gamma L_\gamma/2\overline{Y}$ against y/L_γ (figure 7) indicates that the self-preservation of intensity distributions is a consequence of the self-preservation of the intermittency factor distribution.

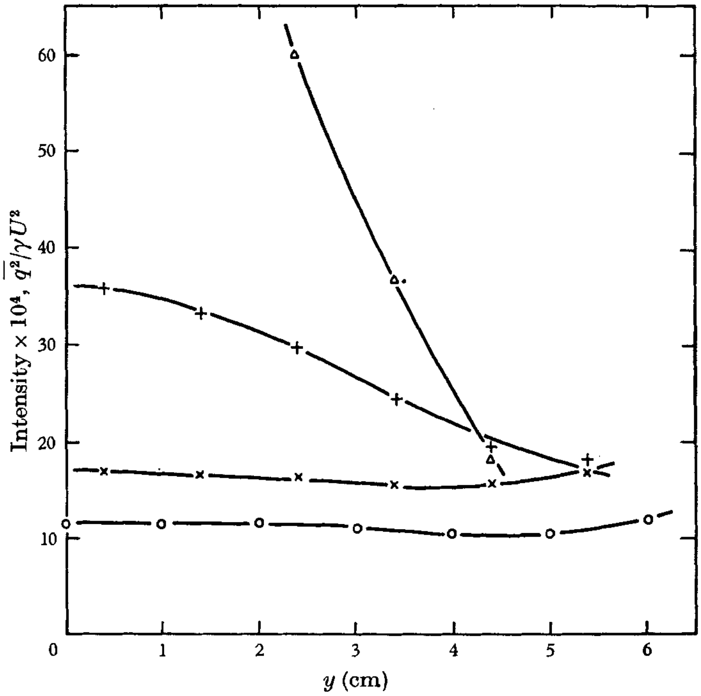

FIGURE 5. Variation of turbulent intensity within the turbulent fluid for the undistorted turbulence. Distance from grid: △, $x = 38$ cm; +, $x = 67$ cm; ×, $x = 104$ cm; ○, $x = 127$ cm.

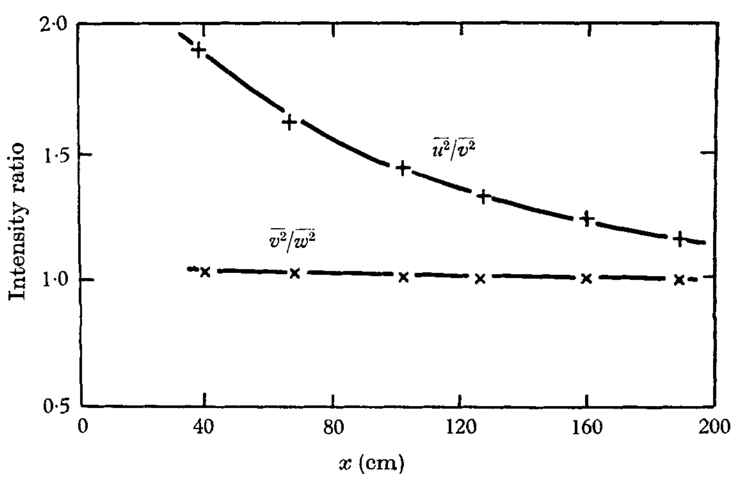

FIGURE 6. Approach to isotropy of the undistorted turbulence on the centre line of the flow.

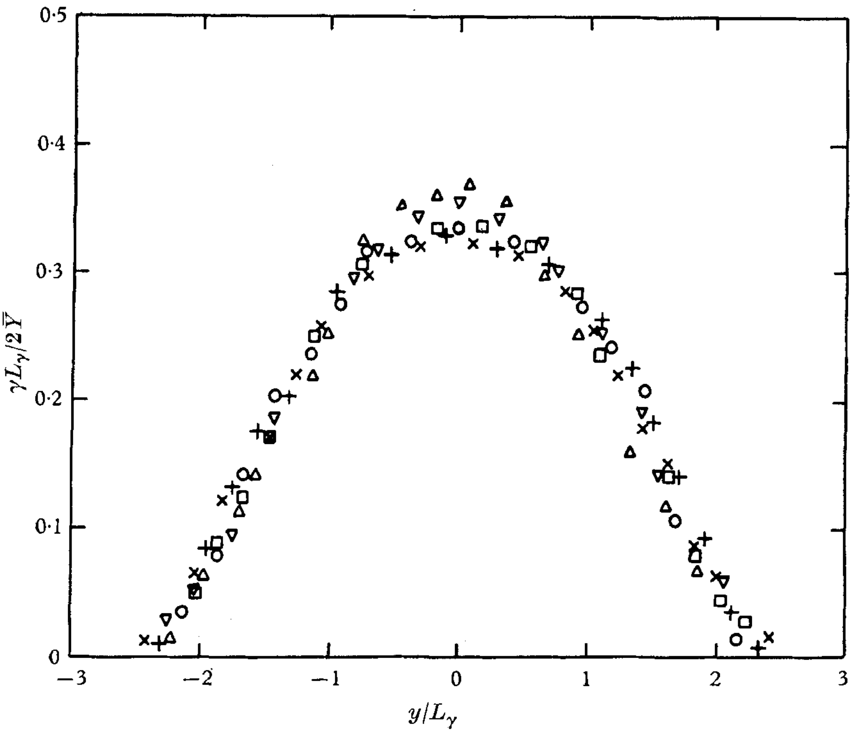

FIGURE 7. Self-preservation of the distribution of intermittency factor. Distance from grid: ○, x = 38 cm; +, x = 67 cm; ×, x = 104 cm; □, x = 127 cm; △, x = 160 cm; ▽, x = 189 cm.

3.2. *The application of a uniform plane strain*

When the second 4 ft. length of wind tunnel working section is replaced by the constant area distorting duct, straining of the flow commences soon after the turbulence has started to contract. The true effect of applying the plane strain on the mean turbulence boundary position and the width of the intermittent zone is ascertained from the variation of $\beta\bar{Y}$ and βL_γ, where β is the strain ratio (figures 3 and 4). The straining has no immediate effect on either the mean boundary position or the width of the intermittent zone. The turbulence continues to contract and about half the ultimate strain ratio β of 2 has been applied before contraction is replaced by spreading. The response of the large eddies to the strain is even slower; almost the end of the distortion has been reached before βL_γ becomes greater than the corresponding value of L_γ for the unstrained turbulence.

The effect of the distortion on the structure of the turbulence is seen in figure 8 which shows the variation of the structure parameters K_1 and K_1' on the centre line of the flow. The behaviour is essentially the same as that recorded by Townsend (1954) during the passage of unbounded homogeneous turbulence through a similar distortion. There is an increase in the anisotropy of both the main body of the turbulence and the dissipating eddies. Across the flow K_1 and K_1' are virtually constant up to the point at which contraction of the turbulence gives

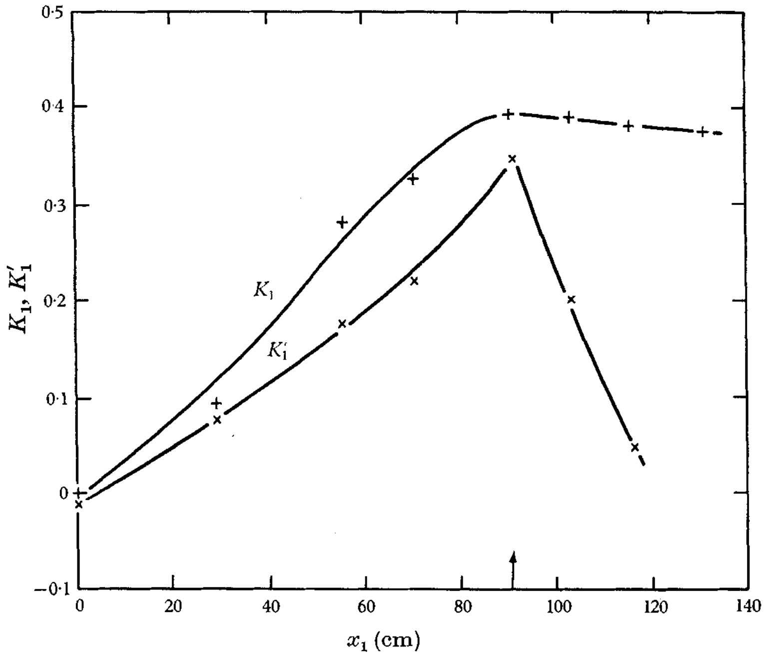

FIGURE 8. Variation of the structure parameters K_1 and K_1' on the centre line of the flow with distance x_1 from the start of the distortion.

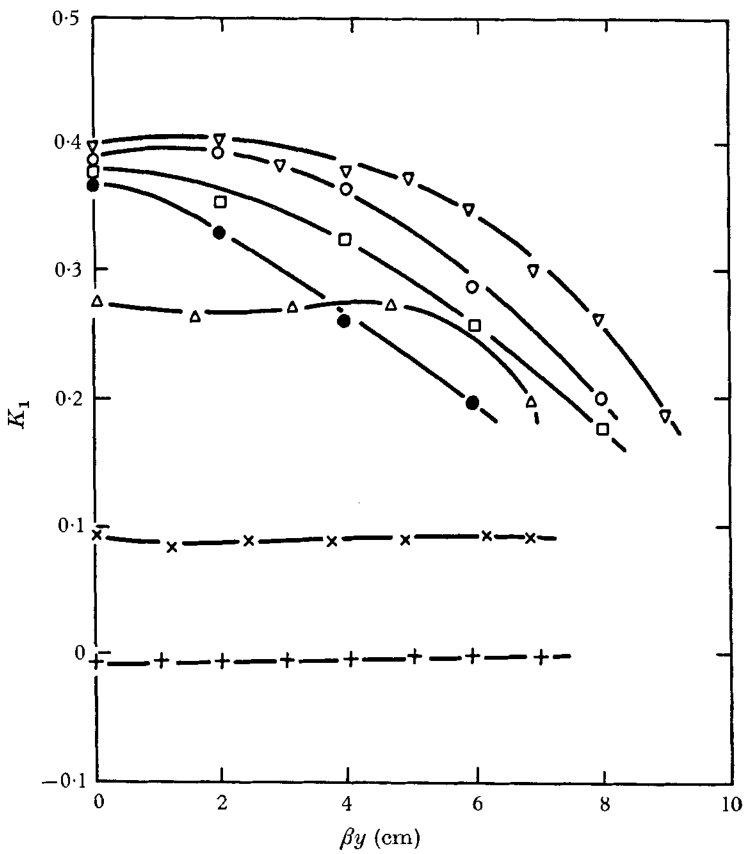

FIGURE 9. Distribution of the structure parameter K_1. Values of strain ratio: $+$, $\beta = 1{\cdot}00$; $\times$, $\beta = 1{\cdot}24$; $\triangle$, $\beta = 1{\cdot}53$; ∇, $\beta = 2{\cdot}00$. Distance x_2 from end of distortion: ∇, $x_2 = 0$ cm; $\bigcirc$, $x_2 = 12{\cdot}5$ cm; $\square$, $x_2 = 25$ cm; $\bullet$, $x_2 = 40$ cm.

way to spreading (figures 9 and 10). Subsequently, the presence of newly created turbulence, which has not been subjected to the full strain, becomes evident, particularly in the outer regions of the flow.

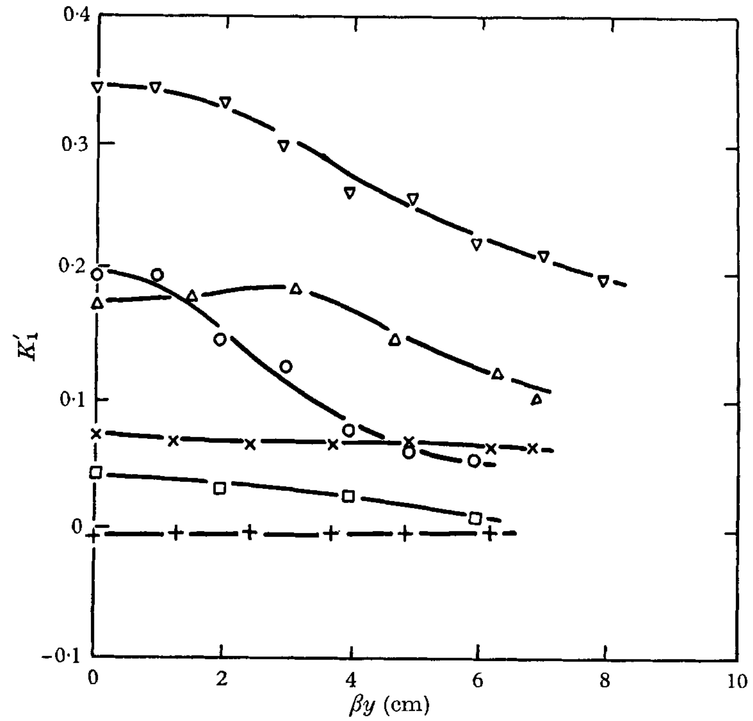

FIGURE 10. Distribution of structure parameter K_1'. Values of strain ratio: $+$, $\beta = 1{\cdot}00$; $\times$, $\beta = 1{\cdot}24$; $\triangle$, $\beta = 1{\cdot}53$; ∇, $\beta = 2{\cdot}00$. Distance x_2 from end of distortion: ∇, $x_2 = 0$ cm; $\bigcirc$, $x_2 = 12{\cdot}5$ cm; $\square$, $x_2 = 25$ cm.

3.3. *The effect of removing the strain*

Following the distortion, the flow enters a length of parallel sided duct, so that the straining suddenly ceases. The effect of removing the strain is seen almost immediately. The turbulence, which had been spreading rapidly at the end of the distortion, quickly reverts to contracting at the same rate which previously prevailed when the turbulence was unstrained (figure 3).

At the end of the distorting duct L_γ, as distinct from βL_γ, happens to have an almost zero rate of change with distance from the grid and when straining ceases L_γ remains practically constant (figure 4).

Again the structure of the turbulence behaves essentially in the same manner as unbounded homogeneous turbulence after the release of strain (Townsend 1954). The variation of K_1 and K_1' on the centre line of the flow shows only a slow trend towards isotropy on the part of the energy containing eddies, while the dissipating eddies show a much more rapid return towards the isotropic state. In the outer regions of the flow, however, K_1 decreases more rapidly than it does on the flow centre line (figure 9).

4. Discussion

4.1. *The contraction of unstrained free turbulence*

In a free shear flow, the transmission of vorticity from the turbulent to the non-turbulent fluid can only take place by the direct action of viscous forces. However, Townsend (1956) has pointed out that the principle of Reynolds number similarity requires the rate of entrainment to be independent of viscosity. The apparent explanation is that once vorticity has been acquired by the non-turbulent fluid it can be increased, independent of the viscosity, by straining in the rate of strain field of the energy containing eddies which constitute the bulk of the turbulent motion.

The contraction of unstrained free turbulence is not inconsistent with the above process. Diffusion of vorticity must still take place at the bounding surface but in the absence of mean flow shear, or of a sufficiently intense rate of strain field due to the energy-containing eddies, the newly created turbulence cannot acquire additional vorticity and is short-lived. In these circumstances, the diffusion process will accelerate the decay of the original turbulence on the other side of the boundary and lead to a contraction in the volume of turbulent fluid.

The contraction process appears to be independent of the degree of anisotropy of the turbulent fluid. The contraction rates of the original unstrained turbulence and the unstrained turbulence which has had its anisotropy increased by plane straining are virtually identical. In view of the relatively small total strain achieved, however, it remains possible that a further increase in anisotropy could have some effect.

It may be noted that the constant contraction rate is associated with substantial variations in the value of the ratio $L_\gamma/\overline{Y}$. This is particularly true in the case of the original unstrained turbulence where L_γ increases as $\overline{Y}$ decreases. The contraction rate would, therefore, seem to be independent of the relative depth of the boundary indentations. Since it might be expected that an increase in surface area of the interface would accelerate the contraction, a fairly flat turbulence front is indicated with little change in surface area. A similar conclusion has been drawn by Corrsin & Kistler (1955) from measurements of turbulence pulse lengths in a turbulent boundary layer. The large eddies which are responsible for the surface indentations display a considerable inertia and their behaviour appears to be strongly dependent on conditions prior to the onset of the contraction process. Grant (1958) has advanced the theory that the large eddies are the product of a stress-releasing mechanism which limits the degree of anisotropy acquired by the turbulence when under strain. If this is true, large eddies cannot arise in the unstrained turbulence and must have their origins in the initial shear flow.

4.2. *The response of unstrained free turbulence to the application and subsequent removal of a plane strain*

When the unstrained turbulence is subjected to a plane strain there is an immediate increase in the anisotropy of both the energy-containing and the energy-dissipating eddies. There is no immediate effect on the contraction process, and it is not until K_1 has attained a value of about 0·2 that the turbulence commences to spread. This delayed response implies that any turbulence created by viscous diffusion across the bounding surface does not gain vorticity directly due to straining by the mean flow, but does so by straining in the rate of strain field of the energy-containing eddies when these have become sufficiently aligned by the mean strain.

A high degree of anisotropy of the energy-containing eddies is not by itself sufficient to promote spreading. This is clearly seen in the rapid reversion to contraction which occurs when the strain is removed, in spite of the turbulence remaining highly anisotropic. A possible explanation has been given by Townsend (1956), namely that the effect of the mean strain is twofold; first to produce a favourable alignment of the energy-containing eddies, and secondly to produce a crowding together of these favoured eddies with a consequent intensification of their rate of strain field.

The response of the large eddies to the distortion is even more delayed than the response of the contraction process at the interface. Apparently the changes in the spreading rate are not due to a simple increase in the surface area of the boundary.

4.3. *Application to free turbulent shear flows*

In a typical free turbulent shear flow, the average velocity difference between the turbulent and non-turbulent flow is small. Townsend (1966) suggests that it does not exceed 5 % of the maximum velocity difference across the flow. In a very general way, therefore, we may regard the bounding surface as being at any moment subjected to the straining conditions appropriate to its instantaneous position relative to the mean velocity profile. It may be expected that the anisotropy of the turbulence adjacent to the bounding surface will be a maximum when the front is located near the point of maximum shear, and that this will be the location most favourable to spreading of the turbulence. On the other hand, the turbulence front will spend an appreciable part of its time in regions of zero mean strain and low anisotropy. Contraction seems likely to take place at the boundaries of any bulges in the front which move near to, or beyond, the limits of the mean velocity variation.

Grant (1958) has observed a lack of spreading in a plane wake during periods of low large eddy activity. During these quiescent periods, the interface will probably be close to the mean boundary position $\overline{Y}$. The comparison in figure 11 of the distribution of intermittency factor, mean velocity, and shear coefficient distribution in a plane wake (Townsend 1949, 1956), indicates that such a position is not particularly favourable to entrainment. An increase in large eddy activity will result in the bounding surface spending more time in locations favourable to spreading. However, the possibility exists that if the interface becomes too highly indented it will be increasingly forced into regions where contraction will prevail and the overall entrainment rate will fall.

The latter view is to some extent supported by the behaviour of a two-dimensional wake when subjected to plane straining (Reynolds 1962; Keffer 1965). There is an initial period of rapid wake expansion which is not sustained. This may be due to the large eddy activity being increased beyond some optimum value.

If contraction of the turbulence does take place in parts of a free turbulent shear flow, difficulties must arise in the use of contaminants for flow visualization, since it would no longer be possible to assume that the boundaries of the contaminant are those of the turbulent fluid.

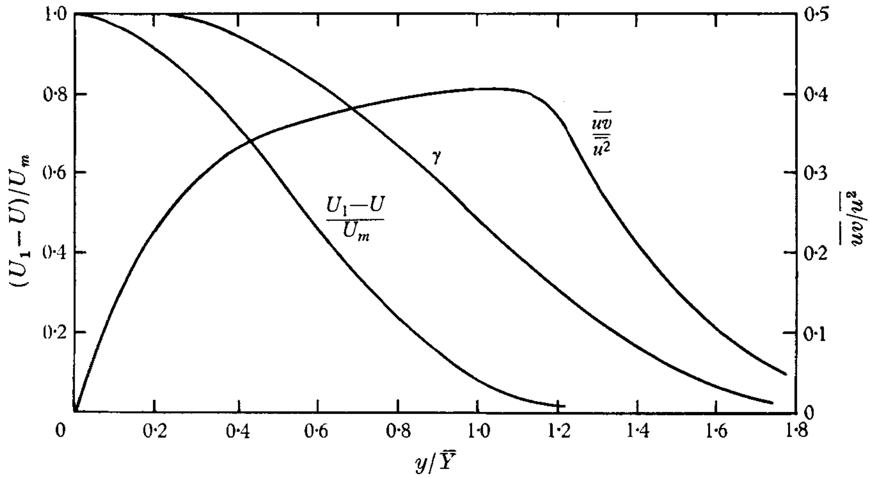

FIGURE 11. Comparison of mean velocity, intermittency factor, and shear coefficient distributions in the plane wake (Townsend 1949, 1956).

5. Conclusions

It is concluded that the rate of advance of the turbulence front in a free turbulent flow depends, not on the local turbulent vorticity, but on the degree of anisotropy of the turbulence and the local mean strain. The mechanism whereby the large eddies increase the entrainment rate is believed to be more complex than a simple increase in surface area of the interface.

REFERENCES

CORRSIN, S. & KISTLER, A. L. 1955 *Nat. Adv. Comm. Aero. Wash. Rept.* no. 1244.
GARTSHORE, I. S. 1966 *J. Fluid Mech.* **24**, 89.
GRANT, H. L. 1958 *J. Fluid Mech.* **4**, 149.
KEFFER, J. F. 1965 *J. Fluid Mech.* **22**, 135.
NAUDASCHER, E. 1965 *J. Fluid Mech.* **22**, 625.
REYNOLDS, A. J. 1962 *J. Fluid Mech.* **13**, 333.
TOWNSEND, A. A. 1949 *Aust. J. Sci. Res.* **2**, 451.
TOWNSEND, A. A. 1954 *Quart. J. Mech. Appl. Math.* **4**, 308.
TOWNSEND, A. A. 1956 *The Structure of Turbulent Shear Flow*. Cambridge University Press.
TOWNSEND, A. A. 1966 *J. Fluid Mech.* **26**, 689.